CONCEPT AND MATHEMATICAL MODELING OF ACOUSTIC EMISSION SOURCE

W. J. Pardee
Science Center, Rockwell International
Thousand Oaks, California 91360

Presently, most acoustic emission applications in nondestructive testing involve placing transducers on a large structure, a bridge or hydrocarbon cracking pressure vessel, then loading the structure and listening for the burst-type acoustic emissions that ensue, and finally using triangulation to locate the flaws whence came those bursts. This technique is very effective at locating defects, and that means that the typical, most studied acoustic emission parameters are just the number of events, the event rate, and to a somewhat lesser extent, the maximum amplitude . It might be the root mean square amplitude, or some other amplitude that characterizes the acoustic emission, or an amplitude distribution, that is, the number greater than some threshold V. These are typical, widely used acoustic emission parameters, and they have been very successful in locating defects. In fact, if they hadn't been, I don't think I would be here today talking about fancier things in acoustic emission.

However, they generally don't tell you all you can hope to find out. For example, you would like to use acoustic emission not only as a nondestructive testing tool to locate defects, but as a monitoring tool with which to warn you when something significant is happening. "Significant" is the key word, because many things going on in that structure generate acoustic waves which may be important. A very important example is crack growth under cyclic loading conditions. The rubbing of two cracked surfaces together generates acoustic emissions copiously, more so than crack propagation, and the crack surface friction generates a very significant signal. It has nothing to do with the length or width of the crack, and you would like to be able to distinguish it from crack growth; presently we can't do that.

To try to do that we must look at more detail in the signals. Instead of looking at what are largely statistical properties of large numbers of signals, we are trying to gain additional information by looking at the individual wave forms, in other words, the shape of a single acoustic emission. You can look at three components of displacement. You can look at them for all time or equivalently, Fourier transform and look at them in frequency space. I might add we don't have infinite media; so this means you look at it on the surface, so that gives you two dimensional wave vectors or a two dimensional distribution in space.

Both Roger Clough and I are talking about the same thing. What we look at is the Fourier transform or perhaps the time domain signal at a particular point in space averaged over a smaller region that is about the size of the detector.

Let me give you a specific example. The largest source of acoustic emission in most aluminum alloys is the fracture of brittle second phase inclusions. These are quite small, perhaps 20 microns or so. Before they fracture they almost certainly vibrate in one of their lowest normal modes. Of course, these modes are damped by their interaction with the medium, but the spatial dependence is determined by the size of the intermetallic particles, and the frequency is determined roughly by the elastic constants of the brittle intermetallic particles. So, if you could Fourier transform and measure both variables, one would expect a peak in ωk space corresponding (in k space) to the size of the particle and something (in frequency space) roughly characterizing its elastic properties. This is something which will give a great deal of fundamental information.

I described our acoustic emission theory in its heuristic and embryonic form a year ago at this meeting; I'm still talking about the same theory, more or less, with some refinements. We have learned a great deal more since then. But I want to review the elements of it and bring you up to date on our progress with it.

The basic picture that we started from is that there are really three important things that determine the frequency spectrum, and these can be varied independently. These are the source, the medium, and the detector. We have absolutely incontrovertible experimental evidence that each of these has a nontrivial influence, under some circumstances, on the spectrum you observe. I'm almost belaboring the obvious, but, of course, the actual details of the source process influences the frequency spectra. Not so obviously, the wave propagation characteristics of the medium affect the frequency spectrum. And, of course, the transducer response affects the frequency spectrum. Because I happened to be challenged about that recently, I brought along a slide (Fig. 1) to show you the difference between a piezoelectric transducer and a capacitor microphone when detecting the same source, which happened to be a fracture of a small silicon carbide grain. Because the piezoelectric transducer gives you a bigger signal, they are very commonly used for a great many acoustic emissions. When you first look at it, the dominant structure you see in a broad band analysis like this is going to be just that due to the response of the transducer.

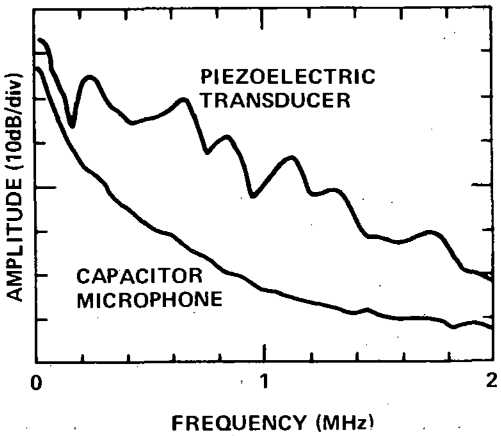

Figure 1. Amplitude responses for a piezoelectric transducer and a capacitor microphone pickup.

To take advantage of these three effects which are conceptually separate--we wrote our mathematics that way--we developed a transfer function formalism which includes all the properties in the medium --the boundary conditions in addition to attenuation and dispersion. We separately describe the source and we have to match that source to the transfer function of the medium and then later put on the transfer function of the transducer. Capacitor microphones, although they are not very sensitive, show a predictable frequency dependence; so, that doesn't complicate things too much. I might add that in the time domain the overall signal is a double convolution integral of these three things, while in the frequency domain it's just a product of these three factors. I think that that is probably the best reason for working in the frequency domain. It's not the only reason, however.

Now, I'm going to describe the theoretical principles of our acoustic emission wave form analysis. These are actually a set of conclusions we have come to by trial and error and, although they are conclusions, they are rather subjective, which is why I call them theoretical principles. They're not so much statements about the way nature works as about the way we work.

The first principle is to observe AE in the far field region. The applications, as I said, are typically to large structures--bridges, pressure vessels, things like that--but the experiments, because of the obvious size limitations, are done on small specimens where you are rather close to the crack. In fact, you are very frequently in the near field region and with complicated geometries. This is an intolerable situation for a theorist, and leads to the second principle, which is to avoid, as much as possible, spurious geometry dependent effects.

As an example, I want to particularly mention the complications concomitant to trying to do detailed model calculations for compact tension specimens. There are two things that come to mind. One is that the actual acoustic emission source with a compact tension specimen is going to be small--not the whole crack, but just a small segment of it--and the actual place where that occurs can be anyplace along the whole crack front. The position varies with crack propagation on a scale which is large compared to both the wave length and the distance of the transducer. Actually, I think that's not an intolerable problem. A greater consequence is the presence of all those boundaries which bias the frequency spectrum.

Let me show you the next slide, (Fig. 2), a very nice experiment done by Lloyd Graham, where a silicon carbide grain is fractured on a plate, which for our purposes may be regarded as infinite. In fact, it was about 6 inches square. The upper curve was obtained at the center of the bottom of the plate. The lower curve was obtained at a remote location on the top surface. That means 4 or 5 cm away, and the place was a half inch thick. As you can see, they are qualitatively the same. There is a significant difference (notice the log scale). The dashed curve is the electronic noise level. I wanted to make this point clear, because if you try to compare with a model calculation or if you try to look at the details and assume that they are valid when extrapolated to a large system, you will err, I believe. I do not want to overemphasize this point, but what one usually looks for is gross qualitative changes. I certainly don't suggest that those gross qualitative changes are going to be affected by these phenomena I just described.

The second point is to avoid spurious geometrical effects like very complicated geometries. But concomitant to that is an obligation not to neglect fundamental geometrical effects. For example, we have to make the measurements on the surface. What we find for the plate is that if you neglect the first surface, you get the wrong answer, an infinite medium does not give the right frequency falloff at high frequencies. You get roughly the right falloff at high frequencies if you include the first surface.

To get quantitative agreement, what we find is that you have to include the second surface. However, the calculations are for an infinite plate and I believe that's reasonable; we seem to be able to get quantitative results, although it's too soon to tell for sure. I think that a system with two parallel surfaces is a pretty good starting place; it's probably a good approximation to many nondestructive testing applications.

The third principle, and this is also even more subjective, is to avoid if possible, explicit normal mode decompositions. There are several reasons: first, it takes a whole lot of them to go to high frequencies. Second, you have to worry about mode conversions and reflections of these modes at the surface. The normal modes are very geometry dependent; they're very sensitive to the details and geometry of your system. I don't believe that the acoustic emission overall wave packet is that sensitive, and to illustrate that point in a very simple system, I considered a vibrating string.

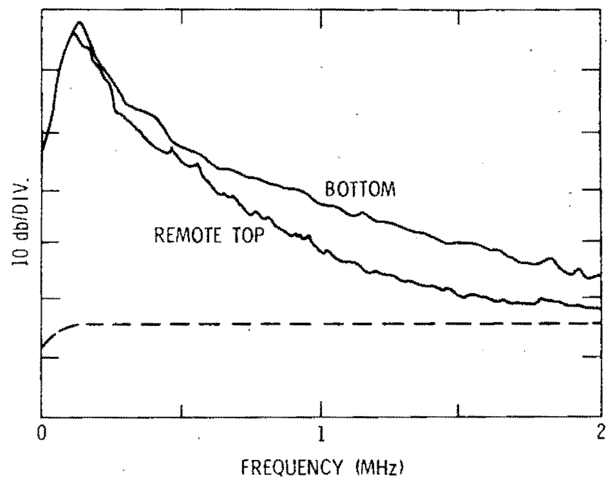

Figure 2. Emission from fracture of silicon carbide grain on a flat plate.

The vibrating string is picked up at t = 0 at each end and released. The exact solution is a square wave propagating between each end. I graphed the first six normal modes (Fig. 3) from the Fourier transform or the Fourier series analysis of this problem; the dotted curve is the sum of those first six. You can see the sum of the first six is not a very good approximation to the actual square wave. This is at a time t = L/4C later where L is the length of the string and C is a velocity. And more importantly, none of the individual normal modes looks at all like the solution. The exact solution, incidentally, is most easily obtained by using the solution for the one dimensional wave equation in the form F(x+vt) + F(x-vt), although both approaches are mathematically rigorously correct. We get far more physical intuition by avoiding the normal modes, if we can do it, but sometimes that's not possible. We were successful for the plate problem, however.

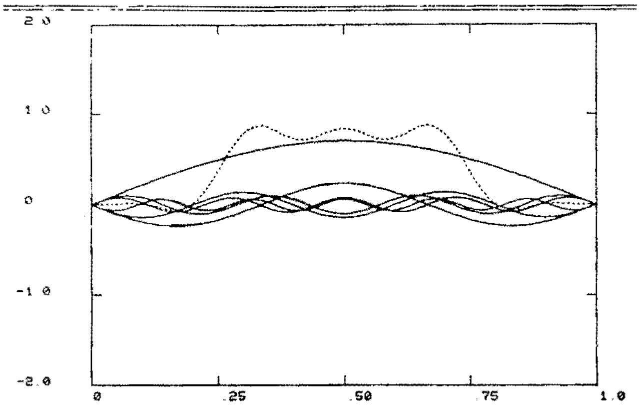

Figure 3. Normal modes of plucked string.

Let me get a little bit more mathematical. Now, the fourth principle is that--and you will see that I have to qualify this a bit later--acoustic emission arises from a sudden change in internal stresses. That certainly is a source of acoustic emission, a major source, and I want to distinguish it from body forces. You don't have real body forces creating these acoustic emissions. So, we try to describe this by nonlinear continuum mechanics, that is

$$\rho\omega^2 U_n + \frac{\partial P_{nm}}{\partial a_n} = 0. \tag{1}$$

P is the mass density, ω the frequency, a_n the Euler variable, U_n is the n component of displacement U; Pnm is the Piola-Kirchhoff stress tensor; the stress tensor modified to include the effects of finite strain and the change in the mass density associated with the finite strain, but it can include a very complicated constitutive relation. It can include a real model for the details of what's happening physically. An example of a physical system described by this equation is a chemical reaction, precipitation of hydrogen out of solution to form a titanium hydride with concomitant expansion of the lattice and excitation of a stress wave. But we have to understand that the waves that we are observing are classical waves and they should be describable by continuum mechanics.

The next point is how can I make this horrible nonlinear equation tractable? As I said, let us work in the far field. I should say that my solution is, in fact, valid in the near field region as long as the near field region is linear. In the far field region it satisfies the same equation except instead of the complicated stress tensor, it's just a linear stress tensor, that is, the linear infinitesimal stress, in which the stress is proportional to the usual infinitesimal strain.

$$\rho\omega^2 U_n + \frac{\partial \sigma_{nm}}{\partial a_m} = 0 \tag{2}$$

So, what I do is add and subtract the linear stress tensor

$$\rho\omega^2 U_n + \frac{\partial \sigma_{nm}}{\partial a_m} - \frac{\partial \sigma_{nm}}{\partial a_m} + \frac{\partial P_{nm}}{\partial a_m} = 0 \tag{3}$$

The first two terms are purely linear and can be treated in a complicated but mathematically tractable fashion. The second two terms include very complicated nonlinear phenomena: plasticity, for example, but they are at least localized. We expect when we get far from the source that it will be a linear problem. The nonlinear effects are concentrated around the source, and for them we neglect the effect of the boundaries on the source itself as a reasonable, although not a universally true assumption. So, we gain a lot of things by this separation.

Last year I presented approximate calculations using substantially this approach for a couple of very simple acoustic emissions. At that time I calculated a transfer function H as an infinite but inadequate sum of some of the normal modes, and we found that at the center of the bottom of the slab we had nearly quantitative agreement: elsewhere, the calculations had only qualitative significance. Our principal mathematical accomplishment this year was to find an exact solution for the plate transfer function which is not a normal mode decomposition. It's an exact treatment of the stress free boundaries on the two surfaces of the slab.

The exact solution of Eqn. 3 in terms of the transfer function H is given by

$$U_n(\omega,\underset{\sim}{r}) = \int dA' \frac{\partial H_{mnab}(\omega,\underset{\sim}{r},\underset{\sim}{r}')}{\partial r_m} n_b(\underset{\sim}{r}')\, U_a(\omega,\underset{\sim}{r}') \tag{4}$$

$$- \frac{1}{\rho\omega^2} \int d^3r' \frac{\partial^3 H_{mnab}(\omega,\underset{\sim}{r},\underset{\sim}{r}')}{\partial r_m \partial r'_b \partial r'_k} (P_{ak}(\omega,\underset{\sim}{r}') - \sigma_{\bar{a}k}(\omega,\underset{\sim}{r}'))$$

The surface integral is over the crack surface, an additional term arising because it does not include the stress free boundary conditions on the surface of the crack. But it still satisfies the same differential equation, and this is an exact statement. U_n is the n component of the displacement field, and that's what will be observed on the surface of the slab. This surface integral is an integral over the freshly created fracture surface of a certain derivative of H multiplied by a unit vector, and U_a is the displacement of the fracture surface. So, we have to understand the displacement of the fracture surface. The second term includes all the nonlinear effects, and what you have to do is solve that problem in the time domain, then Fourier transform it. But that's an infinite medium problem, so you don't have to worry about the boundary conditions. For brittle materials, I think it's a reasonable guess that only the first term contributes. My argument is simply that we probably can neglect nonlinear effects in plastic deformation in very brittle materials: ceramics, for example. It's linear until it fractures, and when it fractures, it's broken.

So, the dominant contribution is just a creation of fresh fracture surface. In fact, you see by looking at the expression that the amplitude you get out is going to be roughly proportional to the fracture surface area created, and it's also proportional to the displacement of that fracture surface area. And if you think about it in more detail and you want to know the instantaneous crack velocity, you realize that you have to look at the details of the way the frequency spectrum falls off.

In practice, the average crack velocity is probably more important than the instantaneous crack velocity, and you would get that with some suitable calibration from the amplitude and the event rate, representing an aposteriori justification for traditional acoustic emission techniques.

I will stop there with a brief comment. As I said, this is a series of conclusions about our approach to acoustic emission. Our plans for the near term are to explore this picture I just described for crack propagation in brittle materials and to do some careful experiments; that is, in geometries which are mathematically tractable and in an effort to quantitatively test this as carefully as possible. And I think we're going to have to iterate a couple of times until the model is good enough to describe quantitatively the experimental spectra. The second thing I want to do is to explore the extent of these near field phenomena so I can tell you something more quantitative about the problems you encounter in the near field region. And the third thing is to explore the polarization dependence; that is, whether it is desirable to look, for example, at transverse displacements or strains and to make--if the theory is reasonably successful--estimates about how high you have to go in frequency to really see the significant features.

DISCUSSION

PROF. JOHN TIEN (Columbia University): Thank you, Bill. Are there any comments?

PROF. GORDON KINO (Stanford University): I have had discussions before, any my viewpoint has changed entirely since I last talked to you, but one of the things I can't get clear in my mind and I wonder about is that when you've got something like a crack developing, it is a very small object. After all, as the crack develops it's a very small area, very small change of length. And typically, say, a piston radiator or any kind of radiator in acoustics tends to get more efficient as the frequency increases. Now, on that basis I would think you would get the noise emission increasing with frequency rather than decreasing with frequency. What is the catch in that argument?

DR. PARDEE: The decrease with frequency that we're seeing here is essentially a kinematic effect due to the fact that a differential equation is second order in time. I think you will see a peak when you get up to the frequency where things are really happening, which I think is perhaps 25 MHz instead of the 2 or 3 MHz that we're looking at. So, we're looking at a kinematic effect in the low frequency end, and you have to look very hard in order to pick out what's happening at much higher frequencies without going up there.

DR. ROBB THOMSON (National Bureau of Standards): Did I hear you say that you did not satisfy the boundary conditions on the crack surface as the crack moves? It seems to me that that would be very important.

DR. PARDEE: It is. I do satisfy all boundary conditions completely. It's just a question of how it's incorporated into the mathematical formalism. It's incorporated aposteriori instead of being built into the Green's function because when I built the Green's function, I don't know where the crack is going to be. The purpose of separation is to avoid the detailed description of the source. So, you have to satisfy them on the crack surface as a separate problem; it's not part of the medium description problem.

PROF. TIEN: Bill, you picked a very simple geometry, silicon carbide grain cracking. I guess the purpose of this entire work of yours is not so much a description of the multifaceted experimental spectra, which people observe, but basically to come up with a physical understanding so they could suggest--suggest what?

DR. PARDEE: Suggest where to look for more sensitive measures, more sensitive diagnostic tools, and how to recognize microscopic characteristics of the source.

PROF. TIEN: So, it's bigger than just some modeling curves?

DR. PARDEE: Certainly!

PROF. MAX WILLIAMS (University of Pittsburgh): What condition do you use to make the crack move?

DR. PARDEE: Presently I have very ad hoc descriptions of the crack propagation. It's not a fundamental theory in the sense that the other is a fundamental theory; it's a phenomenological description of the crack moving. I just say it does move for a certain length of time at a constant velocity.

PROF. WILLIAMS: So, basically--

DR. PARDEE: A very naive picture.

PROF. WILLIAMS: You are prescribing a priori the velocity versus the time of the crack gowth?

PROF. TIEN: You disregard the medium is what you do.

DR. PARDEE: That's right. Then from having prescribed that, one hopes, if one fits the acoustic emission to determine what that velocity is.

PROF. WILLIAMS: There is a difference in the character of the singularities of the crack growth as to whether the crack is running or it initiates and it's tied up with the initial radiation from the crack point; I think you might be leaving out a significant physical factor if you have the crack running a priori to the prescribed velocity.

DR. PARDEE: The correct description--I'm not sure exactly I understand the point you're getting at--but the correct description does require a detailed description of the stresses around the crack tip and their dynamics. So, you need the dynamic stresses around a moving crack which, of course, is a difficult problem by itself.

PROF. WILLIAMS: It's almost impossible.

DR. PARDEE: That's correct. However, I think it's possible to make progress with simpler models. In particular I reaffirm my suggestion that for brittle materials this may not be very important.

PROF. TIEN: One more question, Bill.

DR. WILLIAM A. ELLINGSON (Argonne National Lab): You threw a slide up there rather quickly about a calibration of your transducer. Would you just briefly say how you obtained that curve?

DR. PARDEE: I'm not sure exactly what slide you're talking about. Is it the slide showing the difference between the behavior on the top surface and the bottom surface?

DR. ELLINGSON: Right.

PROF. TIEN: The capacitor one.

DR. ELLINGSON: Yes, the capacitance microphone and apparently the pzt unit.

DR. PARDEE: Well, those were experiments done by Lloyd Graham. I can describe how they were done. You have a plate with a small amount of silicon carbide power on the surface and you have the detector, either the capacitor microphone or the pzt transducer at a remote point on the surface. You grind the silicon carbide powder and use the Hewlett-Packard spectrum analyzer to obtain the resulting spectrum. In fact, it's a composite of many events in that case.

PROF. TIEN: Thank you, Bill.